Time Value of Money

→ **Why Time Value of money :- Why Interest :-**

1. Time Value of money.
2. opportunity cost.
3. liquidity preference.
4. Inflation.
5. Risk factor.

→ **Types of Interest :** →

First Period Amt of C.I
Second Period of Principal

In 1 Year Simple & Compound Int Same

Simple Interest

eg :- Rs 1000 & rate = 10%
time = 3 years.

sol → P = Rs 1000 → Initial Amount.
↳ Principal Amt.

$\Rightarrow \frac{1000 \times 10}{100} \Rightarrow 100$ Rs

$\Rightarrow \frac{1{,}000 \times 10}{100} \Rightarrow 100$ Rs

$\Rightarrow \frac{1000 \times 10}{100} \Rightarrow 100$ Rs

$$S.I = \frac{P \times r \times t}{100}$$

$\Rightarrow \frac{1000 \times 10 \times 3}{100} \Rightarrow 300$ Rs

Amount = 1000 + 300 = 1300 Rs

P + I = A

Compound Interest

eg :- P = Rs 1000 r = 10%
Time = 3 years.

sol → $\frac{1000 \times 10}{100} = 100$ Rs

• After 1 year = 1100 Rs.

→ $\frac{1100 \times 10}{100} = 110$ Rs.

• After 2 year = 1210 Rs.

→ $\frac{1210 \times 10}{100} \Rightarrow 121$

• After 3 year = 1331 Ans

$$Amt = P\left(1 + \frac{r}{100}\right)^n$$

$\Rightarrow 1000\left(1 + \frac{10}{100}\right)^3$

$\Rightarrow 1331$ Ans

Simple Interest

Type 1:- Based on general Formula:-

$$S.I = \frac{P \times r \times T}{100}$$

P = Principal r = rate T = Time

eg:- P = 10,000 r = 5% T = 5 years.

$\Rightarrow S.I = \frac{10,000 \times 5}{100} \times 5 \Rightarrow 2500$

eg:- P = 10,000 r = 5% T = 3 years. 6 months.

$\Rightarrow \frac{10,000 \times 5}{100} \times \frac{7}{2} = 1750$

month ko year me change karna ke lie divided me 12.

eg:- P = 10,000 r = 7 1/2 % Time = 3 year 3 months

$\Rightarrow \frac{10,000 \times 15}{2 \times 100} \times \frac{13}{4} \Rightarrow 2437.5$

eg:- P = 10,000 r = 10% Time = 3 year

$\Rightarrow S.I = \frac{P r t}{100} \Rightarrow \frac{r}{100} = i = P \times i \times t$

calci Trick:-

$10,000 \times .3 \Rightarrow 3,000$

Formula of Amt:-

$$Amt = P\left[1 + \frac{rt}{100}\right]$$

S.I

Date ....

eg:- $P = 10{,}000$ $r = 5\%$ $I = 3000$ time = ?

$S.I \Rightarrow \frac{Prt}{100}$

$\boxed{\frac{S.I \times 100}{Pr} = \text{time}} \Rightarrow \frac{\cancel{3000}^{6} \times 10\cancel{0}}{1\cancel{0{,}000} \times \cancel{5}} \Rightarrow 6$ years.

· **Formula**:-

1. $S.I = \frac{Prt}{100}$
2. $P = \frac{S.I \times 100}{rt}$
3. $r = \frac{S.I \times 100}{Pt}$
4. $t = \frac{S.I \times 100}{Pr}$

4 Values.

- Any three given Next find out

eg:- A sum of money Double itself:-

rate 10%. Time = ?

Sol → Amt = 200 $P = 100$

$S.I = \text{Amt} - P = 200 - 100 = 100$

$\Rightarrow t = \frac{\cancel{100} \times 10\cancel{0}}{\cancel{100} \times 1\cancel{0}} \Rightarrow 10$ years

eg:- S.I on a sum of money is. 25:-

rate = 10% Time = ?

Sol → $P = 100$ $S.I = 25$ $r = 10\%$

$\Rightarrow \text{time} = \frac{25 \times \cancel{100}}{\cancel{100} \times 10} \Rightarrow 2.5$ Years.

Trick:-

(1) Amount [n times]:-

$\Rightarrow \frac{(n-1) \times 100}{r} = t$ $\Rightarrow \frac{(n-1) \times 100}{t} = r$

(2) Simple Int [n time]:

$$\Rightarrow \frac{n \times 100}{r} = t \qquad \Rightarrow \frac{n \times 100}{t} = r$$

eg:- A sum of money become Double in 10 years. In how many years it will treble.

Sol → P $\quad A = 2P$

$S.I = 2P - P = P$

$P = P \times \frac{r}{1\cancel{0}\cancel{0}} \times 1\cancel{0} \Rightarrow r = 10\%$

P $\quad A = 3P$

$S.I = 3P - P = 2P$

$2P = P \times \frac{1\cancel{0}}{1\cancel{0}\cancel{0}} \times t \Rightarrow t = 20$ years.

Shortcut:-

Double in 10 years

treble $\quad t_2 = ?$

n_1 time in t_1 years.
n_2 time in t_2 years. $\Rightarrow \frac{n_1 - 1}{n_2 - 1} = \frac{t_1}{t_2}$

$$\Rightarrow \frac{1}{2} \times \frac{10}{t_2} \Rightarrow t_2 = 20$$

* n times in t years $\quad r = ?$

$$\Rightarrow \frac{(n-1) \times 100}{r} = t \qquad \Rightarrow \frac{(n-1) \times 100}{t} = r$$

Date ....

* S.I. n times in t years r = ?

$\Rightarrow \frac{n \times 100}{t} = r$ $\Rightarrow \frac{n \times 100}{r} = t$

Type-1

- Gape $S.I_2 - SI_1 \Rightarrow \frac{P(r_2 - r_1)t}{100}$
- Gape $S.I_2 - SI_1 \Rightarrow \frac{(P_2 - P_1) r t}{100}$
- Gape $S.I_2 - S.I_1 \Rightarrow \frac{P r (t_2 - t_1)}{100}$

Type 2:- Amt for Two Period is given

eg:- A sum of money becomes 3000 in 3 years & 4000 in 7 years Principal & Rate Find?

Sol → $\left[\begin{array}{ll} A_1 - 3000 & t_1 - 3\text{ year} \\ A_2 - 4000 & t_2 - 7\text{ year} \end{array}\right] \to 4$ years

1000

$\to S.I = 250 \times 3$ year $= 750$

$= P = 3000 - 750 = 2250$

$\to P = 2250$ $S.I = 250$ $t = 1$ $r = ?$

$r = \frac{250 \times 100}{2250 \times 1} \Rightarrow 11.11\%$

P=2250

$\Rightarrow$ 1000 $\left(\begin{array}{l} 3{,}000 \\ 4{,}000 \end{array} \right.$ $\left. \begin{array}{l} 3 \\ 7 \end{array} \right)$ 4

$S.I = 250 \times 3 = 750$ $r \Rightarrow \frac{250 \times 100}{2250} \Rightarrow 11.11\%$

$P = 4{,}000$

eg:- $\frac{1000}{2}$ (6000 → 7,000) 4 years, 6 years) 2

$\Rightarrow 500 \times 4 = 2000$

$r \Rightarrow \frac{500 \times 100}{4000} \Rightarrow 12.5\%$

· If we convert months into years → Divide by 12.

→ 9 months → $9/12 = 3/4$ years.
→ 8 months → $8/12 = 2/3$ years.

⇒ **Shortcut** :- If S.I. same [like of No → 21 level-3]

$\Rightarrow \frac{1}{r_1 t_1} : \frac{1}{r_2 t_2} : \frac{1}{r_3 t_3}$

$\Rightarrow \frac{1}{2 \times 6} : \frac{1}{3 \times 8} : \frac{1}{6 \times 6}$

$\frac{1}{12} : \frac{1}{24} : \frac{1}{36} \Rightarrow \frac{6:3:2}{72}$

$6 : 3 : 2 = 11$) 4000
$24{,}000 : 12{,}000 : 8{,}000 = 44{,}000$

→ **If Amount Same**:-

$\Rightarrow \frac{1}{1+r_1 t_1} : \frac{1}{1+r_2 t_2} : \frac{1}{1+r_3 t_3}$

Date ....

Compound Interest

Type-1 Based on General formula

method-1 C.I

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n$$

eg:- $P = 1000$ $r = 10\%$ time = 3 years.

$$\text{Amount} = 1000\left(1+\frac{10}{100}\right)^3$$

$$= 1000\ (1.1)^3$$

calci → $1.1 \times = = \times 1000 \Rightarrow 1331$

method-2:- Amount =
G.B.C calci = $1000 + 10\% + 10\% + 10\% = 1331$

method-3:-
calci = $1.1 \times = = \times 1000 = 1331$

eg:- Amount = 1331 $r = 10\%$ time = 3 years.

$$\Rightarrow 1331 = P(1.1)^3$$

· calci → $\underbrace{1.1 \times =}_{\text{2 steps}} = (\div = \times)\ 1331 \Rightarrow 1{,}000$

$$\Rightarrow A = P\left(1+\frac{r}{100}\right)^n$$ → Any three given fourth find out.

C.I
eg:- $P = 14000$ $r = 12\%$ time = 3 years.

· Amount $\Rightarrow 14000\left(1+\frac{12}{100}\right)^3 \Rightarrow 19668.92$

calci → $1.12 \times = = \times 14000 \Rightarrow 19668.92$

$P \Rightarrow (1.12)^3 \div = \times 19668.992 \Rightarrow 14{,}000$

eg:- 10,000 $r = 10\%$ time 3 year 6 month

method-I

Amount $= 10{,}000\,(1.1)^3 \Rightarrow 13310$ $\Rightarrow$ 13975.5 Ans

$= 13310 \times \frac{10}{100} \times \frac{1}{2} = 665.5$

method-II

eg:- $P = 10{,}000$ $r = 10\%$ time = 3 years 6 month

3.5 years.

Amount $= 10{,}000\,(1.1)^{3.5}$

calci $\rightarrow$ 1.1 $\sqrt{\ }$ 12 times $- 1 \times 3.5 + 1 =$ (x=) 12 times

$\times 10{,}000 \Rightarrow 13959.508$

eg:- $P = 10{,}000$ $r = 10\%$ time = 3 year.

compount Intrest

Amount $= 1.1 \times = = \times 10{,}000 = 13310.$

Interest $= 13310 - 10{,}000 = 3310.$

$C.I = $ Amount - Principal

$= P\left(1 + \frac{r}{100}\right)^n - P$

$$\boxed{C.I \Rightarrow P\left[\left(1 + \frac{r}{100}\right)^n - 1\right]} \Rightarrow 10{,}000 \times (1.1)^3 - 1$$

$\Rightarrow 3310$

eg:- $P = 15000$ $r = 12\%$ time = 4 years.

$C.I \Rightarrow \underbrace{(1.12) \times =}_{2\text{ step}} = = -1 \times 15000 = 8602.79$

Date ....

If C.I given :- $8602.79 = P\left[(1.12)^4 - 1\right]$

calci Trick:-

$\underbrace{1.12\times = }_{2\ step}$ $= = -1$ $\div = \times$ 8602.79

$\Rightarrow 15,000$ Ans

eg:- $P = 1000$ $r = 12\%$ time = 2 years.

yearly :- 1 Jan 2021 $P = 1000$ $r = 12\%$ $t = 1$ year

Amt $= 120 + 1000 = 1120$

1 Jan 2020 $P = 1120$ $r = 12\%$ $t = 1$ year

Amt $= 134.4 + 1120 = 1254.4$

Half yearly :- $P = 1000$ $r = 12\%$ time = 2 year

1 Jan 2021 $P = 1000$ $r = 12\%$ $t = 1/2$ year

30 June 21 $\Rightarrow 1000 \times \frac{\cancel{12}^{6}}{100} \times \frac{1}{\cancel{2}} \Rightarrow 60$

$\Rightarrow 1\cancel{000} \times 6/\cancel{100} \Rightarrow 60$ Rs.

- Amt after 6 months $\Rightarrow 1000 + 60 = 1060$

1 July 2021 $P = 1060$ $r = 6\%$

2021 ⇒ 31 Dec $1060 \times 6/100 = 63.6$

- Amt after 1 year $\Rightarrow 1060 + 63.6 = 1123.6$

1 Jan 2022 $P = 1123.6$ $r = 6\%$

30 June 201 $\Rightarrow 1123.6 \times 6/100 = 67.416$

- Amt after 1 year 6 month $\Rightarrow 1123.6 + 67.416$

$\Rightarrow 1191.016.$

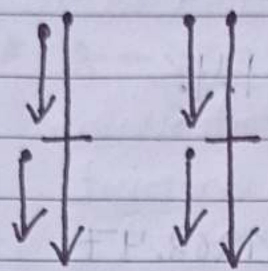
1 July 2022 $P = 1191.016$ $\quad r = 6\%$

31 Dec 2020 $= 1191.016 \times 10/100 = 71.46$

• Amt After 2 years $\Rightarrow 1191.016 + 71.46 \Rightarrow 1262.47$

$P = 1000$ $\quad r = 12\%$ $\quad$ Time = 2 year

$\frac{r}{2} = 6\%$



1 year 2 year

$$\text{Amt} = P\left(1 + \frac{r}{100}\right)^{n} \Rightarrow P\left(1 + \frac{r}{200}\right)^{2n}$$

$$\therefore 1000\left(1 + \frac{6}{100}\right)^{4} = (1.06)^{4} \times 1000$$

calci $\rightarrow 1.06 \times = \,= = \times 1000 \Rightarrow \underline{\underline{1262.47}}$ Ans

quarterly :- $P = 1000$ $\quad r = 12\%$ $\quad$ time = 1 year

1 Jan 2021 $P = 1000$ $\quad r = 3\%$

31 mar 2021 $= 1000 \times 3/100 = 30$

1 April 2021 $P = 1030$ $\quad r = 3\%$

30 Jun 2021 $= 1030 \times 3/100 = 30.9$

1 July 2021 $P = 1060.9$ $\quad r = 3\%$

30 sep 2021 $= 1060.9 \times 3/100 = 31.82$

1 oct 2021 $P = 1092.72$ $\quad r = 3\%$

31 Dec 2021 $= 1092.72 \times 3/100 = 32.78$

Amount of 1 year $\Rightarrow$ 1125.5

Date ....

$P = 1000 \quad r = 12\% \quad time = 1\ years.$

$Amount = P\left(1+\frac{r}{400}\right)^{4n} \Rightarrow 1000\left(1+\frac{3}{100}\right)^{4}$

$\Rightarrow (1.03)\times = = = = \times 1000 \Rightarrow 1125.5.$

* $P = 1000 \quad r = 12\% \quad time - 2 year.$

Yearly :- $Amt = 1000\left(1+\frac{12}{100}\right)^{2} \Rightarrow 1254.4.$

Half yearly :- $Amt = \underbrace{(1.06)\times =}_{2\ time} = = \times 1000 \Rightarrow 1262.47.$

quartelly :- $Amt = \underbrace{(1.03)\times =}_{2\ times} = = = = \times 1000 \Rightarrow 1266.77.$

monthly :- $\underbrace{(1.01)^{24}\times =}_{2\ times}$ = $= \times 1000 \Rightarrow 1269.73.$

* $A = 13310 \quad P = 10{,}000 \quad time\ 3 year \quad Rate = ?$

IBPS

$\Rightarrow A = P\left(1+\frac{r}{100}\right)^{n}$

$\Rightarrow \frac{A}{P} = \left(1+\frac{r}{100}\right)^{n} \rightarrow \left(\frac{A}{P}\right)^{\frac{1}{n}} = \left(1+\frac{r}{100}\right)$

$$\boxed{\Rightarrow \left[\left(\frac{A}{P}\right)^{\frac{1}{n}} - 1\right] 100 = r}$$

$\Rightarrow \frac{13310}{10{,}000} = \left(1+\frac{r}{100}\right)^{3} \Rightarrow 1.331 = \left(1+\frac{r}{100}\right)^{3}$

$\Rightarrow (1.331)^{1/3} = 1+\frac{r}{100}$

$\Rightarrow \left[(1.331)^{\frac{1}{3}} - 1\right] \times 100 = r$ $\Rightarrow \sqrt[n]{A} = A^{\frac{1}{n}}$

$\Rightarrow$ 1.331 $\sqrt{}$ 12 times − 1 ÷ 3 + 1 = (x=) 12 times

$r = 10\%$

A $\sqrt{}$ 12 times − 1 ÷ n + 1 = (x=) 12 times

* $A = 13310$ $P = 10{,}000$ $r = 10\%$ time = ?

$A = P\left(1 + \frac{r}{100}\right)^n = P(1+i)^n$ $\frac{r}{100} = i$

$13310 = 10{,}000\left(1 + \frac{10}{100}\right)^n$

$\frac{13310}{10{,}000} = (1.1)^n$ $\Rightarrow 1.331 = (1.1)^n$

$\frac{A}{P} = \left(1 + \frac{r}{100}\right)^n$

(i) by B.C

(ii) log

method :- 1

$\underbrace{1.1 \times =}_{2\ \text{step}} = 1.331$

method :- 2

by. B. C

(a) 4 (b) 3 (c) 5 (d) None

method :- 3 $1.331 = (1.1)^n$

Taking log both sides.

$\log 1.331 = n \log(1.1)$

$n = \frac{\log 1.331}{\log 1.1} \Rightarrow \frac{0.124175}{0.0413903971} \Rightarrow 3$ years.

log 1.331 | log 1.1

$\Rightarrow$ 1.331 $\sqrt{}$ 12 times − 1 × 227695 = 0.124175

$\Rightarrow$ 1.1 $\sqrt{}$ 12 times − 1 × 227695 = 0.41390 3971

Date ....

$\Rightarrow A = P\left(1 + \frac{x}{100}\right)^n \Rightarrow$ 4 unknown
Any three given

* $P = 10,000$ $x = 10\%$ $n = 3$ $A = 13310$

$\downarrow$ Initial Value $\downarrow$

Principal
Present Value
Sum
Amount Due.

$\downarrow$ Final Value $\downarrow$

Future Value
Amount.

Final Value = $\boxed{\text{Initial Value}\left(1 + \frac{\text{rate}}{100}\right)^{\text{time Period}}}$

100	110	121
10%	10%	10%
10	11	12.1
$\boxed{110}$	$\boxed{121}$	$\boxed{1331}$

Population Growth Rate 10%
1,00,000

$\Rightarrow$ Growth Rate = Birth rate - Death rate

$\Rightarrow \frac{39}{1000} - \frac{19}{1000} = \frac{20}{1000}$

$\frac{2}{100} = 2$ per %

$F.V = 1,00,000\left(1 + \frac{10}{100}\right)^3$

$\Rightarrow 1.1x = = x\,1,00,000 = 133100$

$\Rightarrow 13310 - 1,00,000 = 33100$

Depreciation :- Asset = 10,00,000 Dep rate = 10%.
After 3 year.

1 Jan 2022
10,00,000 - 10% = 1,00,000

31 Dec 2022
9,00,000 Value of Asset.

1 Jan 2023
9,00,000 - 10% = 90,000

31 Dec 2023
8,10,000 Value of Asset.

1 Jan 2024
8,10,000 - 10% = 81,000

31 Dec 2024
7,29,000 Value of Asset.

$$\Rightarrow P - \frac{Pr}{100} = \boxed{P\left(1-\frac{r}{100}\right)}^{F.V} - P\left(1-\frac{r}{100}\right)\cdot\frac{r}{100}$$

$$\Rightarrow P\left(1-\frac{r}{100}\right)\left[1-\frac{r}{100}\right]\left[1-\frac{r}{100}\right]$$

$$\Rightarrow P\left(1-\frac{r}{100}\right)^n$$

$$\Rightarrow F.V = I.V\left(1-\frac{r}{100}\right)^n$$

IV → cost of Asset.
r → Rate of Depreciation.
n → useful life.
FV → Scrap Value.

$$\Rightarrow S.V = 10,00,000\left(1-\frac{10}{100}\right)^3$$

$$= (1.1)^3 = 7,29,000$$

$$(.9)x = x \cdot 10,00,000$$

Date ....

Shortcut

Type2 :- C.I.

Double Time $\left(\frac{69}{r} + .35\right)$

$r = 2\%$ $\quad \frac{69}{2} + .35 \Rightarrow 34.85$ Approximate

C.I

Triple Time $\left(\frac{111.444}{r} + .35\right)$

$r = 2\%$ $\quad \frac{111.444}{2} + .35 \Rightarrow 56$ years.

* A sum of money double itself in 8 years. In how many year it will become 8 times of itself.

$\Rightarrow 8P = P\left(1 + \frac{r}{100}\right)^{n_2} \rightarrow$ Find out

given :- $2P \cdot 1 = P\left(1 + \frac{r}{100}\right)^{8}$

$\Rightarrow 2 = \left(1 + \frac{r}{100}\right)^{8}$

$2^3 = \left(1 + \frac{r}{100}\right)^{n_2}$

$\left(\left(1 + \frac{r}{100}\right)^{8}\right)^{3} = \left(1 + \frac{r}{100}\right)^{n_2}$

$\left(1 + \frac{r}{100}\right)^{24} = \left(1 + \frac{r}{100}\right)^{n_2}$

$n_2 = 24$

* Shortcut

A sum of money Double itself in 8 years ... times eight time

$\Rightarrow$ 2 $\quad$ 3 ——— 8

$8 = 2^3$

24 years.

Spiral

* Difference between S.I & C.I is Rs 60 rate = 10% time = 2y.
Principal = ?

* C.I - S.I = Difference

- $P\left[\left(1+\frac{r}{100}\right)^n - 1\right] - \frac{Prt}{100} = \text{Difference}$

- $P\left[\left(1+\frac{10}{100}\right)^2 - 1\right] - \frac{P\times 10\times 2}{100} = 60$

- $P\left[(1.1)^2 - 1 - .2\right] = 60$

calci ⇒ $1.1 \times = -1 - .2 \div = \times 2$ 60 ⇒ $P = 6000$

Shortcut

- gap 2 years   Sum ? Difference ? rate = given

$$\boxed{\Rightarrow \text{Sum} = \text{Difference}\left(\frac{100}{r}\right)^2}$$

⇒ $d = 60$. $r = 10$ $\text{Sum} = 60\left(\frac{10\cancel{0}}{1\cancel{0}}\right)^2 = 6000$

- gap 3 years

$$\boxed{\Rightarrow \text{Sum} = \text{Difference}\left(\frac{100}{r}\right)^2\left(\frac{100}{300+r}\right)}$$

Date ....

eg:- $P = 10{,}000$ $r = 10\%$ time = 3 year Diff.

$\Rightarrow 10{,}000 = \text{Diff}\left(\frac{10\cancel{0}}{1\cancel{0}}\right)^2\left(\frac{100}{300+10}\right) = \frac{10{,}000}{310}$

$\Rightarrow C.I - S.I = d$

$\rightarrow P\left[\left(1+\frac{r}{100}\right)^n - 1\right] - \frac{Prt}{100} = d$

$\rightarrow P\left[\left(1+\frac{r}{100}\right)^n - 1 - \frac{rt}{100}\right] = d$

$\Rightarrow 10{,}000\left[(1.1)^3 - 1 - .3\right] = d$

calci = $1.1 \times = = -1 - .3 \times 10{,}000 = 310$

Type 4:- Amount for Two period is given consecutive years.

eg:- $A = 6500$ 4 years

7000 5 years. Principal or Rate $\Rightarrow$?

$\Rightarrow P = 65000$ $A = 7000$

• $S.I = 7000 - 6500 = 500$

$\rightarrow$ rate $= \frac{500 \times 100}{6500} = 7.6923\%$ Ans

$\rightarrow 6500 = P(1.076923)^4 = P = 4832.5$ Ans.

Spiral

eg:- 6200 3 years
7400 4 years Principal of rate = ?

- $r = \frac{1200 \times 100}{6200} = 19.35\%$
- $6200 = P(1.1935)^3$

$\Rightarrow P = 3646.9$ Ans

eg:- 6000 A_1 4 years n_1
7200 A_2 6 years n_2

$\Rightarrow A_1 = P\left(1+\frac{r}{100}\right)^{n_1}$

$\Rightarrow A_2 = P\left(1+\frac{r}{100}\right)^{n_2}$ $\Rightarrow$ $\frac{6000 = \cancel{P}\left(1+\frac{r}{100}\right)^4}{7200 = \cancel{P}\left(1+\frac{r}{100}\right)^6}$

$= \left(1+\frac{r}{100}\right)^2 = \frac{7200}{6000} = \left(1+\frac{r}{100}\right) = (1.2)^{1/2}$

$\frac{r}{100} = (1.2)^{1/2} - 1 \Rightarrow r = 9.54\%$ Ans

$\Rightarrow 6000 = P(1.0954)^4 \Rightarrow P = 4167.3.5$ Ans

Type-5 Effective Rate of Interest

(Nominal rate)

eg:- $P = 10{,}000$ $r = 12\%$ time 1 year.

- C.I $\Rightarrow$ yearly $\rightarrow 10{,}000[(1.12)^1 - 1] \Rightarrow 1200$
  - $\Rightarrow$ Halfyearly $\rightarrow 10{,}000[(1.06)^2 - 1] \Rightarrow 1236$
  - $\Rightarrow$ quarterly $\rightarrow 10{,}000[(1.03)^4 - 1] \Rightarrow 1255.0881.$
  - $\Rightarrow$ monthly $\rightarrow 10{,}000[(1.01)^{12} - 1] \Rightarrow 1268.2503.$

Date ....

• S.I ⇒ yearly = 10,000 × 12/100 × 1 ⇒ 1200

* effective rate of Independent of Principal

⇒ effective rate = $(1+i)^n - 1$ → yearly

⇒ effective rate = $(1+i)^n - 1$ → Half yearly.

↳ $i = r/200 \mid n = 2t$

⇒ effective rate = $i = r/400 \mid n = 4$. → quarterly.

⇒ effective rate = $i = r/1200 \mid n = 12$ → monthly.

eg:- $r = 6\%$ P.A quarterly

effective rate = $\left[\left(1 + \frac{6}{400}\right)^4 - 1\right] \times 100$ ⇒ rate = 6.136%.

⇒ Half yearly ⇒ $A = P\left(1 + \frac{r}{100}\right)^n$

⇒ After 2 years ⇒ $P\left(1 + \frac{2r}{100}\right)^{n/2}$

⇒ quarterly ⇒ $P\left(1 + \frac{r}{400}\right)^{4n}$

⇒ After 5 year ⇒ $P\left(1 + \frac{5r}{100}\right)^{n/5}$

Spiral

1st 2nd 3rd

eg:- P = 10,000 r_1 = 10% 12% 16%

Amt after 3 years.

calci :- 10,000 + 10% + 12% + 16% = 14291.2

method-II :-

calci:- 10,000 (1.1)(1.12)(1.16)

⇒ 14291.2

eg:- P = 10,000 r_1 = 10% r_2 = 15% r_3 = 20%

3 years Next 5 year next 2 year

Amt after 10 years.

calci:- 10,000 + 10% + 10% + 10% + 15% + 15% + 15% 15% +

15% + 20% + 20% ⇒ 38550

method-II:- $10,000\,(1.1)^3\,(1.15)^5\,(1.2)^2$

⇒ 38550

Date ....

Annuity ⇒ equal Instalment.
[Fix period]

eg:- Rent of House.
:- life Insurance Premium.
:- Payment of Housing loan.
:- Vehicle loan etc.
:- sinking Fund Instalment.
:- Pension.

eg:- 1 Jan 2022	5000	1 Jan 22	5000
1 Feb 2022	5000	1 Feb 22	6000
1 April 2022	5000	1 March 22	5000
1 June 2022	5000	1 April 22	5000
Not Annuity		Not Annuity	

Compound Interest

Present Value → Principal

Future Value → Amount

$$\text{Amount} = \text{Principal}\left(1+\frac{r}{100}\right)^n$$

$$F.V = PV\,(1+i)^n$$

$$\Rightarrow \boxed{\frac{F.V}{(1+i)^n} = PV}$$

Case-1 Annuity [P.V] ordinary | Total = 4000 | r = 10% |

eg:- I = 909.09
II = 826.44
III = 751.31

F.V	1000	1000	1000	1000
	End of 1 year	2y	3y	4y

IV ⇒ 683.01

⇒ 3169.05

Spiral

Annuity Due [Pv] (Case-II) $r = 10\%$ (4000)
[Immediate]

1000 → 1000, 1000 → $\frac{1000}{1.1}$, 1000 → $\frac{1000}{(1.1)^2}$, 1000 → $\frac{1000}{(1.1)^3}$

I → 1000
II → 909.09
III → 826.44
IV → 751.31
Total = 3486.84

F.V ordinary (Case-III) $r = 10\%$ $\boxed{4000}$

1000 ($P.V_1$), 1000 (PV_2), 1000 (PV_3), 1000 (PV_4)

$F.V_1 = 1000(1.1)^3$ $F.V_2 = 1000(1.1)^2$
$F.V_3 = 1000(1.1)$

After 4 years.
I → 1331
II → 1210
III → 1100
IV → 1000
Total = 4641

F.V Due [Immedites] (Case-IV) $r = 10\%$ (4000)

PV_1 1000 → FV_1 $1000(1.1)^4$
PV_2 1000 → FV_2 $1000(1.1)^3$
PV_3 1000 → FV_3 $1000(1.1)^2$
PV_4 1000 → FV_4 $1000(1.1)$

⇒ After 4 year.
I → 1464.1
II → 1331
III → 1210
IV → 1100
Total = 5105.1

Date ....

· **Formula of case III** :- G.P series [Common Ratio]

$\Rightarrow 1000 + 1000(1.1) + 1000(1.1)^2 + 1000(1.1)^3$

G.P $\Rightarrow$ $\boxed{S_n = \frac{a(r^n - 1)}{r-1}}$

$\Rightarrow S_n = 1000 \frac{(1.1)^4 - 1}{1.1 - 1}$

$\Rightarrow 4641$ Ans

$a = 1000$
$r = 1.1$
$n = 4$

$\rightarrow \frac{A[(1+i)^n - 1]}{1+i-1}$

F.V $\Rightarrow$ $\boxed{\frac{A[(1+i)^n - 1]}{i}}$

· **Formula of case IV** :-

$\Rightarrow 1000(1.1) + 1000(1.1)^2 + 1000(1.1)^3 + 1000(1.1)^4$

$\Rightarrow (1.1)[1000 + 1000(1.1) + 1000(1.1)^2 + 1000(1.1)^3]$

F.V $\Rightarrow$ $\boxed{\frac{A((1+i)^n - 1)}{i}(1+i)}$

· **Formula of case I** :- G.P series

$\Rightarrow \frac{1000}{(1.1)} + \frac{1000}{(1.1)^2} + \frac{1000}{(1.1)^3} + \frac{1000}{(1.1)^4}$

$\boxed{S_n = \frac{a(1 + r^n)}{1 - r}}$ $\quad a = \frac{1000}{1.1} \quad r = \frac{1}{1.1} \quad n = 4$

$$S_n = \frac{\frac{1000}{1.1}\left[1 - \frac{1}{(1.1)^n}\right]}{1 - 1/1.1}$$

$$\Rightarrow \frac{\frac{1000}{1.1}\left[\frac{(1.1)^n - 1}{(1.1)^n}\right]}{\frac{1.1-1}{1.1}} \qquad \frac{1.1-1}{1.1} = \frac{.1}{1.1}$$

$$\Rightarrow \frac{1000}{\cancel{1.1}}\left[\frac{(1.1)^n-1}{(1.1)^n}\right] \times \frac{\cancel{1.1}}{.1}$$

$$P.V \Rightarrow \boxed{\frac{A[(1+i)^n-1]}{i(1+i)^n}}$$

$$\Rightarrow \frac{1000[(1.1)^4-1)]}{.1(1.1)^4} \Rightarrow 3169.86 \text{ Ans}$$

· **Formula of case II** :-

$$\Rightarrow 1000 + \frac{1000}{1.1} + \frac{1000}{(1.1)^2} + \frac{1000}{(1.1)^3}$$

$$P.V \Rightarrow \boxed{\frac{A[(1+i)^n-1]}{i(1+i)^n}(1+i)} \Rightarrow 3169.86 \times (1.1)$$

$$\Rightarrow 3486.846$$

(1) **Annuity** ⇒ equal Instalment, every, each

(2)

(loan) → Present Value
- Regular (or) ordinary → First Payment end of year. [Nothing mention]
- Pre (or) Immediate → First Payment in Begining

Consuming Now

(Investment) → Future Value
- Regular (or) ordinary
- Pre (or) Immedia

Date ....

P.V

$\Rightarrow A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] \Longrightarrow PV = \left[\frac{-A[(1+i)^n-1)]}{i}\right]$

F.V

$\Rightarrow A\left[\frac{(1+i)^n-1}{i}\right] \Longrightarrow FV = \left[\frac{(1+i)^n-1}{i}\right]A$

Immediate → (1+i) multiply.

calci Trick :- $\frac{-1000((1.1)^{-4}-1)}{.1}$

$\underbrace{(1.1)\div =}_{1\ step} = = = -1 \div .1 \times 1000$

$=3169.86$

Pre $= 3486.846$

$\frac{1000((1.1)^{4}-1)}{.1}$

$\underbrace{1.1\times =}_{2\ step} = = -1 \div .1 \times 1000$

$= 4641$

Pre $= 5105.1$

Application of Annuity

1. Sinking Fund :-
2. leasing :-

lessor (owner) (of Assets)

lessee (user)

MIND MAP

RITU JINDAL

TIME VALUE OF MONEY

Interest

Simple Interest

- $S.I = \frac{Prt}{100}$
  - $P = \frac{S.I \times 100}{rt}$
  - $r = \frac{S.I \times 100}{Pt}$
  - $t = \frac{S.I \times 100}{Pr}$
- Difference only Two Values
  - $S_1 - S_2 = \frac{(P_1 - P_2)rt}{100}$
  - $S_1 - S_2 = \frac{P(r_1 - r_2)t}{100}$
  - $S_1 - S_2 = \frac{Pr(t_1 - t_2)}{100}$

Simple Interest Types of Questions

- Amount n times
  - $\frac{(n-1) \times 100}{r} = t$
  - or
  - $\frac{(n-1) \times 100}{t} = r$
- S.I. n times
  - $\frac{(n) \times 100}{r} = t$
  - or
  - $\frac{(n) \times 100}{t} = r$
- n_1 times in t_1 years, n_2 times in t_2 years
  - $\frac{n_1 - 1}{n_2 - 1} = \frac{t_1}{t_2}$
- Divide Sum of Money Into Parts
  - $\frac{1}{r_1 t_1} : \frac{1}{r_2 t_2} :$
  - S.I. Same
- Amount for two Periods
  - 1 year S.I. is given
  - $1000 \left[\begin{matrix} 5000 & 4 \\ 6000 & 8 \end{matrix} \right] 4$

Conversion

- Year into Month
- Month into Year
- Days into Month
- Month into Days

C.I.

- Divide Sum of Money into parts Amount Same
  - x
  - $\frac{1}{(1+i_1)^{n_1}} : \frac{1}{(1+i_2)^{n_2}}$
- n_1 times t_1 years, n_2 times ?
  - Eg. 2 times 4 years, 64 times = ?
  - 2^6 → $6 \times 4 = 24$

Compound Interest

- Yearly → $A = P(1+i)^n$
- Half Yearly → $\frac{i}{2}, 2n$
- Quarterly → $\frac{i}{4}, 4n$
- Monthly → $\frac{i}{12}, 12n$

Types of Questions

- Sum Double
  - $\frac{69}{r} + .35$
  - or $\frac{72}{r}$
- Sum Triple
  - $\frac{111.444}{r} + .35$
  - or $\frac{114}{r}$
- More than 3 years Proper
- Amount for Two Periods

Application of C.I.

- Population → $FV = IV(1+i)^n$
- Depreciation → $FV = IV(1-i)^n$
- Effective Rate → $r = [(1+i)^n - 1]$

Difference between C.I and S.I

- gap 2 years → $Sum = diff \left(\frac{100}{r}\right)^2$
- gap 3 years → $Sum = diff \left(\frac{100}{r}\right)^2 \left(\frac{100}{300+r}\right)$
- gap More than 3 years → $C.I - S.I = P((1+i)^n - 1 - it)$

MIND MAP
ANNUITY

RITU JINDAL

- **Future Value**
  - Ordinary ⇒ $FV = A\left[\frac{(1+i)^n - 1}{i}\right]$
  - Immediate or Due ⇒ $FV = A\left[\frac{(1+i)^n - 1}{i}\right](1+i)$
- **Present Value**
  - Ordinary ⇒ $P.V = A\left[\frac{1-(1+i)^{-n}}{i}\right]$
  - Immediate or Due ⇒ $P.V = A\left[\frac{1-(1+i)^{-n}}{i}\right](1+i)$

Compound Interest

⇓

$A = P(1+i)^n$

⇓

$P = \frac{A}{(1+i)^n}$ or

$i = \left(\frac{A}{P}\right)^{\frac{1}{n}} - 1$ or

for $n \Rightarrow \left(\frac{A}{P}\right) = (1+i)^n$

either GBC or taking log

Application of Annuity

- Sinking Fund ⇒ F.V.
- Leasing ⇒ P.V
- Capital Expenditure ⇒ P.V
- Valuation of Bond ⇒ P.V
- Perpetuity
  - Forever ⇒ $= \frac{R}{i}$
  - Growing Perpetuity ⇒ $\frac{R}{i-g}$

- **Net Present Value**
  - $N > 0$ ⇒ Accept
  - $N = 0$ ⇒ Depend
  - $N < 0$ ⇒ Reject
- **Compound Annual Growth Rate** ⇒ $rate = \left(\frac{V(t_n)}{V(t_0)}\right)^{\frac{1}{t_n - t_0}} - 1$

NPV → P.V of Cash inflow − P.V of Cash outflow